

# C.U.SHAH UNIVERSITY

## Summer Examination-2018

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT2

Branch: B.Tech (All)

Semester : 2

Date : 25/04/2018

Time : 10:30 To 01:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1**                      **Attempt the following questions:** **(14)**

- a) The value of  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^7 x \, dx$  is  
 (A)  $\frac{32\pi}{35}$  (B)  $\frac{32}{35}$  (C) zero (D)  $\frac{16}{35}$
- b) If  $f_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$ , then  $(f_n + f_{n-2})$  is equal to \_\_\_\_\_.  
 (A)  $\frac{1}{n}$  (B)  $\frac{1}{n-1}$  (C)  $\frac{n}{n-1}$  (D)  $\frac{n-1}{n}$
- c)  $B(1,1) =$  \_\_\_\_\_  
 (A) 1 (B) 0 (C) 1/2 (D) none of these
- d)  $\frac{1}{4} \sqrt{\frac{3}{4}} =$  \_\_\_\_\_  
 (A)  $\frac{\pi}{\sqrt{2}}$  (B)  $\pi\sqrt{2}$  (C)  $\sqrt{2\pi}$  (D) none of these
- e)  $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-2\sin^2 \theta}}$  is equal to  
 (A)  $\frac{1}{\sqrt{2}} E\left(\frac{1}{\sqrt{2}}\right)$  (B)  $\frac{1}{2} K\left(\frac{1}{\sqrt{2}}\right)$  (C)  $\frac{1}{\sqrt{2}} K\left(\frac{1}{\sqrt{2}}\right)$  (D)  $\frac{1}{2} E\left(\frac{1}{\sqrt{2}}\right)$
- f)  $\operatorname{erf}(-x)$  is  
 (A) an odd function (B) an even function (C) neither odd nor even function  
 (D) none of these
- g) If the power of  $x$  are even, then the curve is symmetrical about  
 (A) X-axis (B) Y-axis (C) about both X and Y axes (D) none of these



- h) The curve  $x^3 + y^3 = 3axy$  represent  
 (A) Cissoid of Diocle (B) Witch of Agnesi (C) Strophoid  
 (D) Folium of Descartes
- i)  $\int_0^a \int_0^{\sqrt{a^2-y^2}} dx dy$  is equal to  
 (A)  $\pi a^2$  (B)  $\frac{\pi a^2}{2}$  (C)  $\frac{\pi a^2}{4}$  (D) none of these
- j) On converting into polar coordinates  $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} dx dy$  is equal to  
 (A)  $\int_0^{\pi/2} \int_0^{2a\cos\theta} r dr d\theta$  (B)  $\int_0^{\pi/2} \int_0^{2a\sin\theta} r dr d\theta$  (C)  $\int_0^{\pi/2} \int_0^{2a\sin\theta} r dr d\theta$  (D) none of these
- k) The degree and order of the differential equation of all parabolas whose axis is x-axis are  
 (A) 2, 1 (B) 1, 2 (C) 3, 2 (D) none of these
- l) The solution of the equation  $(x+y)^2 \frac{dy}{dx} = a^2$  is  
 (A)  $(x+y) - a \tan^{-1}\left(\frac{x+y}{a}\right) = x+c$  (B)  $(x+y)a \tan x = x+c$   
 (C)  $(x+y) - a \cos a = x+c$  (D) none of these
- m) The infinite series  $1+r+r^2+\dots+r^{n-1}+\dots$  is divergent if  
 (A)  $|r| < 1$  (B)  $|r| > 1$  (C)  $r \geq 1$  (D)  $r = -1$
- n) The sum of the series  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$  is  
 (A)  $\log 2$  (B) zero (C) infinite (D) none of these

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions (14)**

a) Using reduction formula prove that  $\int_0^{\pi} x \sin^7 x \cos^4 x dx = \frac{16\pi}{1155}$  (5)

b) Evaluate:  $\int_0^{\infty} x^4 e^{-x^4} dx$  (5)

c) Evaluate:  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$  (4)

**Q-3 Attempt all questions (14)**

a) Show that  $\int_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{\sqrt{\pi}}{n} \cdot \frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n} + \frac{1}{2}\right)}$  (5)

b) Solve:  $xdy - ydx = \sqrt{x^2 + y^2} dx$  (5)



c) Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2 + 1}$ . (4)

**Q-4 Attempt all questions** (14)

a) Evaluate  $\int_0^a \int_0^{\sqrt{a^2 - y^2}} (x^2 + y^2) dx dy$  by changing into polar coordinates. (5)

b) Test for convergence the series  $\sum_{n=1}^{\infty} \frac{[(n+1)x]^n}{n^{n+1}}$ . (5)

c) Evaluate:  $\int_0^{\frac{\pi}{6}} \cos^6 3\theta \sin^2 6\theta d\theta$  (4)

**Q-5 Attempt all questions** (14)

a) Solve:  $\frac{dy}{dx} + 2y \tan x = \sin x$  given that  $y = 0$  when  $x = \frac{\pi}{3}$ . (5)

b) Change the order of integration in the integral  $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy dx$  and evaluate it. (5)

c) Prove that  $B(m, n) = B(m, n+1) + B(m+1, n)$  (4)

**Q-6 Attempt all questions** (14)

a) Test the convergence of the series  $\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$  (5)

b) Using reduction formula prove that  $\int_0^a x^5 (2a^2 - x^2)^{-3} dx = \frac{1}{2} \left( \log 2 - \frac{1}{2} \right)$ . (5)

c) Solve:  $(y^2 e^{xy^2} + 4x^3) dx + (2xye^{xy^2} - 3y^2) dy = 0$  (4)

**Q-7 Attempt all questions** (14)

a) Trace the curve  $y^2(2a - x) = x^3$ . (5)

b) Find the volume of the solid generated by the revolution of the loop of the curve  $x(x^2 + y^2) = a(x^2 - y^2)$  about the  $x$  - axis. (5)

c) Evaluate:  $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\cos x}}$  (4)

**Q-8 Attempt all questions** (14)

a) For small values of  $x$ , show that  $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \left( x - \frac{x^3}{1!3} + \frac{x^5}{2!5} - \frac{x^7}{3!7} + \dots \right)$ . (5)

b) Trace the curve  $r^2 = a^2 \cos 2\theta$ . (5)

c) Find the length of the arc of the curve  $y = \log \sec x$  from  $x = 0$  to  $x = \frac{\pi}{3}$ . (4)

